

Finite size scaling in the solar wind magnetic field energy density as seen by WIND

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Abstract. Statistical properties of the interplanetary magnetic field fluctuations can provide an important insight into the solar wind turbulent cascade. Recently, analysis of the Probability Density Functions (PDF) of the velocity and magnetic field fluctuations has shown that these exhibit non-Gaussian properties on small time scales while large scale features appear to be uncorrelated. Here we apply the finite size scaling technique to explore the scaling of the magnetic field energy density fluctuations as seen by WIND. We find a single scaling sufficient to collapse the curves over the entire investigated range. The rescaled PDF follow a non Gaussian distribution with asymptotic behavior well described by the Gamma distribution arising from a finite range Lévy walk. Such mono scaling suggests that a Fokker-Planck approach can be applied to study the PDF dynamics. These results strongly suggest the existence of a common, nonlinear process on the time scale up to 26 hours.

1. Introduction

Statistical properties of the interplanetary magnetic field (IMF) fluctuations are a topic of considerable interest in space research. The subject is closely related to energy transport and acceleration processes in the solar wind [Tu and Marsch, 1995; Burlaga, 2001; Goldstein and Roberts, 1999; Milovanov and Zelenyi, 1998]. An approach is to consider MHD turbulence as the process responsible for observed statistical features of the IMF fluctuations such as intermittency [Tu and Marsch, 1995; Burlaga, 2001]. Single point measurements can not uniquely determine the existence of turbulence [Frisch, 1995] and ideally one needs to construct a structure function from a range of spatial locations in the fluid. However, data taken over long intervals in the solar wind is routinely single point and can yield strongly suggestive, if not definitive results, and it is this type of data that we treat here.

A hallmark of statistical intermittency is the presence of large deviations from the average value on different scales. An increased probability of finding such large deviations is manifested in the departure of the PDF from Gaussian toward a leptokurtic distribution [Bohr *et al.*, 1998; Sornette,

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2000] (see also [van Atta and Park, 1972]). It is well established that such intermittency can be caused by the fluctuations in the rate of energy transfer of the turbulent cascade [Neart *et al.*, 1995] as suggested in Kolmogorov's 1962 theory [Kolmogorov, 1962]. Following Kolmogorov's ideas Castaing proposed an empirical model for the PDF of velocity differences [Castaing *et al.*, 1990]. Given that: 1) for constant energy transfer rate ϵ , the fluctuating quantity has a Gaussian distribution and 2) the width of the Gaussian has a log-normal distribution it was shown that the resultant Castaing distribution gives a good fit to the velocity difference PDFs in laboratory fluid turbulence experiments. In the case of the solar wind, one can obtain a good fit to the PDF of velocity and magnetic field fluctuations by this method [Sorriso-Valvo *et al.*, 1999, 2001] or can consider the fluctuations about the mean value [Padhye *et al.*, 2001], but as we shall see this technique can not be applied over the entire dynamic range of the magnetic field. Fitting a curve to a real PDF over its full range is rather difficult due to the large statistical errors present for the less frequently occurring large fluctuations. In practice one has to discard significant amount of data in these tails of the PDFs and work only around the center of the distribution curves. For example, in Sorriso-Valvo *et al.* [1999] all events larger than 3 times the standard deviation of the original sample were neglected. The PDF rescaling method used in this paper is free of that limitation as we do not fit a specific distribution to the data. Instead, we extract the scaling properties of the fluctuations directly from the data. Importantly the scaling for the entire dynamic range of the investigated PDFs is obtained from the statistics of the smallest fluctuations. The scaling is thus derived using points in the center of the distribution (with very small statistical errors) and then applied to the PDFs as a whole.

We find that the PDF of fluctuations in magnetic energy density is mono scaling. The corresponding collapse of the rescaled PDF curves reveals the time scales over which similar physical processes occur and also confirms any correlations suggested by the inverse power-law form of the power spectra. Moreover, it quantifies the asymptotic behavior of the distribution of the fluctuations, which is essential for constraining turbulence models [Bohr *et al.*, 1998]. Given the scaling exponent we can then use a Fokker-Planck approach to model the statistical behavior of the time series.

The WIND solar wind magnetic field [Lepping *et al.*, 1995] dataset [Freeman *et al.*, 2000a, b] comprises over 1 million, 46 second averaged samples from January 1995 to December 1998 inclusive, calculated from the WIND spacecraft key parameter database. The selection criteria for solar wind data was given by the component of the spacecraft position vector along the Earth-Sun line, $X > 0$, and the vector magnitude, $R > 30$ RE. The data set includes intervals of both slow and fast speed streams. Similarly to other satellite measurements short gaps in WIND data file were present. To minimize the errors caused by such incomplete measurements we omitted any intervals where the gap was larger than 15%. The original data were not averaged nor detrended. The data is not evenly sampled but there are two dominant sampling frequencies: 1/46 Hz and 1/92 Hz. We use sampling frequency f_s of 1/46 as our base and treat other temporal resolutions as gaps when the accuracy requires it ($\tau \leq 92$ seconds).

2. Scaling of the fluctuation PDF

Differencing techniques date back to the 1920s when Richardson conveyed the image of the turbulent cascade and proposed the velocity difference as an ideal field for statistical studies [Frisch, 1995]. In MHD turbulence the differencing method can also be applied to magnetic field magnitude measurements. Here, however, we will first investigate the scaling properties of the magnetic energy density fluctuations given by $B^2(t)$. Let $B(t)$ represent the time series of the magnetic field magnitude and $B^2(t)$ its corresponding energy density. A set of time series $\delta B^2(t, \tau)$ is obtained for each value of time lag τ :

$$\delta B^2(t, \tau) = B^2(t + \tau) - B^2(t) \quad (1)$$

The PDF $P(\delta B^2, \tau)$ is then calculated for each time series $\delta B^2(t, \tau)$. Figure 1 shows the resultant unscaled PDFs of the magnetic field energy density fluctuations for different values of the time lag $\tau = 46 \times 2^N$ seconds where $N = 0, 1, 2, \dots, 17$ giving a range of time scales between 46 seconds and about 70 days. We plot the PDF for all values of δB^2 for which there are at least 150 samples per bin.

A generic one parameter rescaling method (the finite size scaling approach, see e.g. Sornette [2000]; Mantegna and Stanley [1995, 2000]) is applied to these PDFs. We first assume that the distributions can be described by a stable, symmetric law (a Gaussian or Lévy [Bardou et al., 2002]) within a finite scaling range. We use the peaks of the PDFs to obtain the scaling exponent s . This is important as the peaks are the most accurate parts of the distributions. Figure 2 shows these PDFs rescaled with respect to their $\sigma(\tau)$ (see e.g. [Castaing et al., 1990; van Atta and Park, 1972]) and we see that these also do not collapse onto a single curve.

Figure 3 shows the peaks $P(0, \tau)$ of the unscaled PDFs (shown in Fig. 1) plotted versus τ on log-log axes. We see that there is a range of τ up to 26 hours for which $P(0, \tau)$ is well described by a power law τ^{-s} with index $s = 0.42 \pm 0.02$. This break in the scaling at $\tau \sim 26$ hours is consistent with that found previously (see e.g. Burlaga [2001]; Sorriso-Valvo et al. [1999, 2001]). Within this range we now attempt to collapse the entire unscaled PDF shown in Fig. 1 onto a single master curve using the following change of variables:

$$P(\delta B^2, \tau) = \tau^{-s} P_s(\delta B^2 \tau^{-s}, \tau) \quad (2)$$

If the initial assumption of the stable and symmetric distributions was correct a single parameter rescaling, given by equation (2), for a mono-fractal process, would give a perfect collapse of PDFs on all scales. A self-similar Brownian walk is a good example of the process where such collapse can be observed (see e.g. Sornette [2000]). Figure 4 shows the result of the above one parameter rescaling applied to the unscaled PDF of IMF energy density fluctuations shown in Fig 1. We see that the rescaling procedure (2) using the value of the exponent s of the peaks $P(0, \tau)$, as shown in Fig 3, gives good collapse of the curves onto a single common functional form for the entire range of the data.

The successful rescaling of the PDFs now reveals some key features of the processes governing the magnetic field energy density fluctuations. In particular, as we will show below, one can quantify an asymptotic behavior of the rescaled PDFs. A single parameter rescaling strongly suggests that the resultant distribution is stable and as such should con-

verge to Gaussian or Lévy distribution for large fluctuations [Sornette, 2000]. The leptokurtic character of the PDFs is still present after the rescaling indicating that the Lévy distribution is more appropriate to model the functional form of the curves. For a system where B^2 has an upper, physical or instrumental, limit such a distribution is truncated. Importantly, truncated Lévy distributions are still 1) stable and 2) go over to Gaussian for sufficiently large time lag τ [Sornette, 2000]. Calculation of the Hurst exponent for the B^2 time series, by direct examination of the $\sigma^2 \propto \tau^{2H}$ relation as well as the “growth of range” algorithm, confirms such a limit does exist - we find a mean exponent $H = 0.28 \pm 0.02$ up to 26 hours beyond which the variance of the signal saturates. We then fit a truncated Lévy distribution to the large δB^2 asymptotes of the rescaled PDFs. This Gamma distribution is of the form [see Sornette, 2000]:

$$P(\delta B^2) = C_{\pm} \exp\left(-\frac{|\delta B^2|}{\Delta}\right) |\delta B^2|^{-(1+\frac{1}{\alpha})} \quad (3)$$

Two dashed lines on Fig. 4 show (displaced for clarity) the Gamma distribution of the large energy density fluctuations. This best fit of (3) to each of the tails of the rescaled PDFs is obtained for the power law index $\alpha \sim 0.66$ and parameters $C_- = 0.9$, $C_+ = 1$ and $\Delta = 100$. The maximum time scale of about 26 hours beyond which the scaling no longer holds is in good agreement with previous results [Sorriso-Valvo *et al.*, 1999, 2001], (see also Burlaga [2001] and references therein).

It has been found previously [Burlaga, 2001] that the magnetic field magnitude fluctuations are not self-similar but rather multi-fractal. For such processes the scaling derived from $P(0, \tau)$ would not be expected to rescale the entire PDF as above. To verify this we repeated the rescaling procedure for magnetic field magnitude differences $\delta B(t, \tau) = B(t + \tau) - B(t)$. Figure 5 shows the result of one parameter rescaling applied to the PDFs of the magnetic field magnitude fluctuations. The scaling procedure is satisfactory only up to 3 standard deviations of the original sample. This confirms the results of Sorriso-Valvo *et al.* [1999, 2001] where a two parameter Castaing fit to values within 3 standard deviations of the original sample yields scaling in one parameter and weak variation in the other. Attempts to improve the collapse by using information in the tails (values $|\delta B| > 3\sigma$) would introduce a significant error in the estimation of the scaling exponent s . However, the difference in scaling between the PDFs of δB^2 and δB seen here may simply arise from the long range correlations present in the magnetic field magnitude measurements. This follows since:

$$(\delta B)^2 = B^2(t + \tau) - B^2(t) - 2(B(t)B(t + \tau) - B^2(t)) \quad (4)$$

The first two terms, $B^2(t + \tau) - B^2(t)$ are the fluctuation of the magnetic energy density and as we have seen this quantity is mono-scaling. The last term, which includes $B(t)B(t + \tau)$, will however be significant when the autocorrelation function of the IMF magnitude time series is non negligible.

The Fokker-Planck equation provides an important link between statistical studies and the dynamical approach expressed by the Langevin equation [Sornette, 2000]. The mono-scaling of the magnetic field energy density PDF, together with the finite value of the samples’ variance, indicates that a Fokker-Planck approach can be used to study

the dynamics of the unscaled PDF $P(\delta B^2, t)$ in time and with respect to the coordinate δB^2 [van Kampen, 1992]. Recently a fractional Fokker-Planck equation has been obtained that can be used to treat Lévy walk PDF dynamics [Schertzer *et al.*, 1995]. Alternatively, the anomalous diffusion of the PDF, consistent with truncated Lévy walk, can be obtained by introduction of a functional dependence of the diffusion coefficient on the “spatial” coordinate (in our case δB^2). One can then write a Fokker-Planck equation for the evolution of the PDF as:

$$\frac{\partial P}{\partial t} = \nabla_{\beta}(P\gamma(\delta B^2)) + \nabla_{\beta}^2 DP \quad (5)$$

where $P \equiv P(\delta B^2, t)$ is a distribution function, γ is called the friction coefficient and $D \equiv D(\delta B^2)$ is a diffusion coefficient which in this case depends on the magnetic energy density fluctuations. The differential operator $\nabla_{\beta} =$ should be understood as:

$$\nabla_{\beta} = \frac{\partial}{\partial(\delta B^2)} \quad (6)$$

Such a functional dependence of the diffusion coefficient on δB^2 leads to the anomalous diffusion resulting in the departure from the well known Einstein law relating variance of the signal and the time increments *Sornette* [2000].

3. Summary

In this paper we have applied the generic finite size scaling method to study the scaling for the magnetic field energy density and magnitude fluctuations of the IMF using WIND MFI data. This yields the scaling properties of the solar wind magnetic field energy density fluctuations that do not assume a particular turbulent model. Instead, we use the smallest (and best represented statistically) fluctuations to obtain a single parameter that gives the scaling exponent sufficient to rescale the 46 seconds averaged data over its entire dynamic range in δB^2 . The simplest explanation suggested by these results is that the IMF energy density fluctuations are governed by a common physical process that is self-similar on the temporal scale up to $\tau \sim 26$ hours. The rescaled PDF follows a non-Gaussian distribution with an asymptotic behavior well described by the Gamma distribution arising from a finite range Lévy walk. Such rescaling also indicates that a Fokker-Planck approach can be used to study the evolution (in δB^2 space) of the PDF. A similar rescaling procedure applied to magnetic field magnitude fluctuations confirms more complex, perhaps multi-fractal, character of the underlying dynamics in agreement with previous results.

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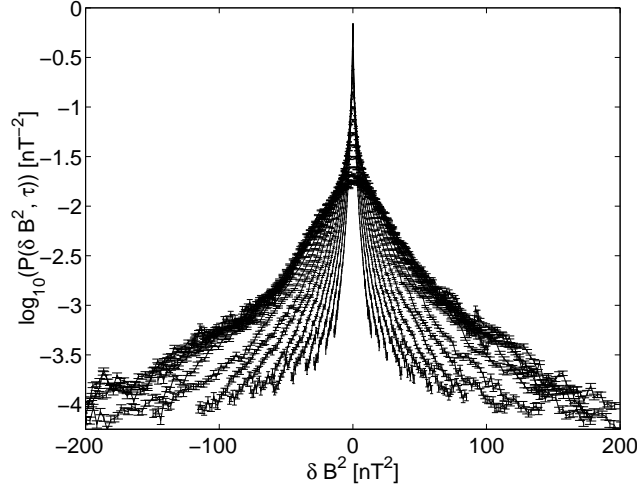


Figure 1. Unscaled PDFs of the IMF energy density fluctuations. Time lag $\tau = 2^k \times 46\text{s}$, where $k = 0, 1, 2, \dots, 17$. Standard deviation increases with τ . Error bars represent 3σ intervals assuming Gaussian distribution of points within each bin.

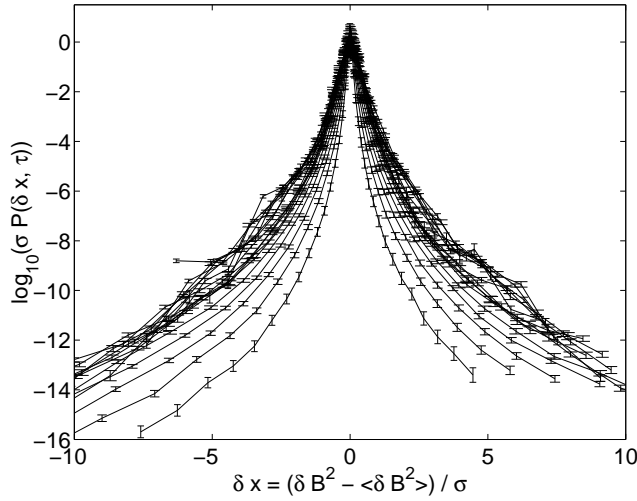


Figure 2. PDFs rescaled with respect to their standard deviation $\sigma(\tau)$. Shown curves correspond to the scaling region from figure 3 i.e., τ between 46 seconds and 26 hours. Rescaled PDFs show no satisfactory collapse. Error bars as in figure 1.

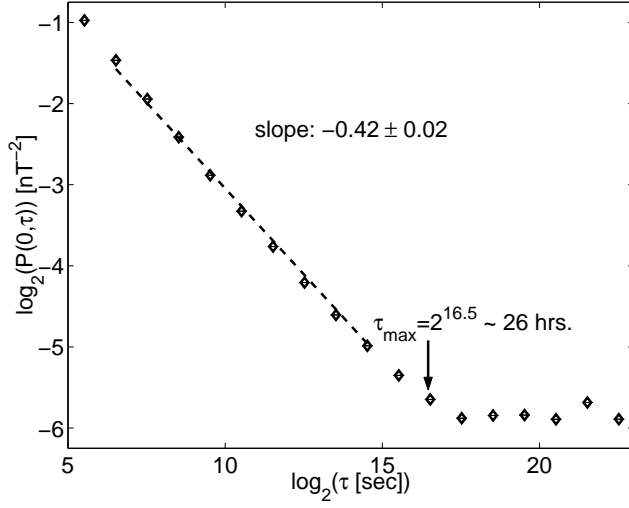


Figure 3. Scaling of peaks $P(0, \tau)$ of the IMF energy density fluctuations PDFs. Slope of the line is fitted with 95% confidence interval. Scaling extends from $\tau = 46$ seconds to about $\tau = 26$ hours. Error bars as in figure 1.

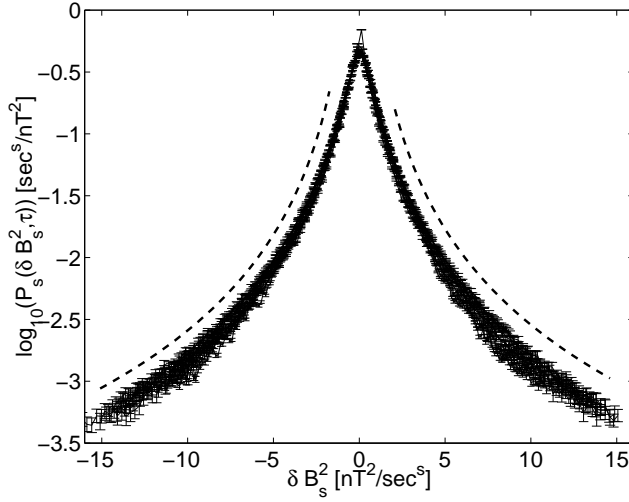


Figure 4. One parameter rescaling of the magnetic field energy density PDFs. The curves shown correspond to τ between 46 seconds and 26 hours. Dashed lines represent the Gamma distribution of the large fluctuations for the PDF of $\tau = 2^4$. Vertical offset was added for clarity. Error bars as in figure 1.

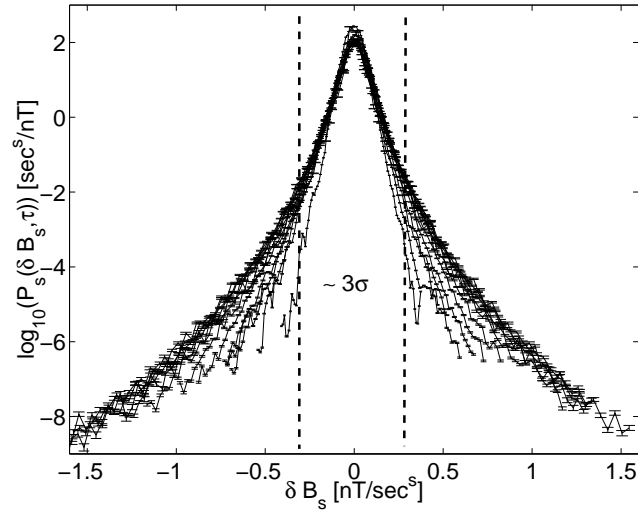


Figure 5. One parameter rescaling of PDFs of IMF fluctuations. Shown curves correspond to τ between 46 seconds and 26 hours. Note the satisfactory collapse to about 3 standard deviations of the original sample and poor collapse of the tails. Error bars as in figure 1.